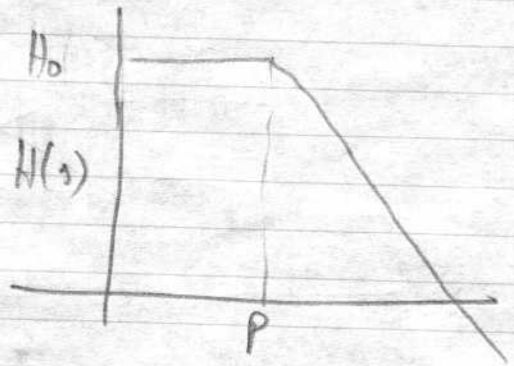
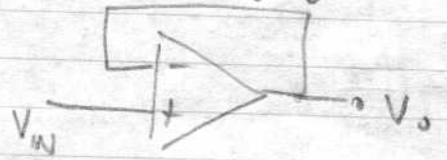
SINGLE POLE SYSTEM IN UNITY GAIN FEEDBACK STEP RESPONSE

Let the system be



$$H(s) = \frac{H_0}{1 + s/p}$$

putting it in unity gain mode:



$$H(s)(V_i - V_o) = V_o$$

$$H(s)V_i = V_o(1 + H(s))$$

$$V_o = V_i \frac{H(s)}{1 + H(s)} = V_i \frac{H_0}{1 + s/p + H_0}$$

Step input $V_i = \frac{V_I}{s}$

$$\therefore V_o = V_I \frac{H_0}{s(1 + s/p + H_0)} = \frac{A}{s} + \frac{B}{s + p(1 + H_0)}$$

$$V_I H_0 p = A(s + p(1 + H_0)) + Bs$$

$$s = 0 \quad \therefore A = \frac{V_I H_0 p}{p(1 + H_0)} = V_I \frac{H_0}{1 + H_0}$$

$$s = -p(1 + H_0) \quad \therefore B = -\frac{V_I H_0 p}{p(1 + H_0)} = -\frac{V_I H_0}{1 + H_0}$$

$$V_o(s) = V_I \frac{H_o}{(1+H_o)} \frac{1}{s} - \frac{V_I H_o}{1+H_o} \frac{1}{s+p(1+H_o)}$$

$$V_o(s) = V_I \frac{H_o}{1+H_o} \left[\frac{1}{s} - \frac{1}{s+p(1+H_o)} \right]$$

$$V_o(t) = V_I \frac{H_o}{1+H_o} \left[1 - e^{-p(1+H_o)t} \right]$$

$\therefore$ time constant of step response

$$\tau = \frac{1}{p(1+H_o)}$$

Intuitively also in feedback the pole moves out & in unity gain it is at $p(1+H_o)$ hence time constant decreases. In general case

$$\tau = \frac{1}{p(1+H_o/G)}$$

where G is the closed loop gain approximation considering $G << H_o$

as $G \rightarrow H_o$ the equation will loose accuracy obviously since for open loop $\tau = \frac{1}{p}$.